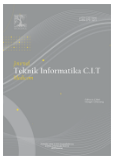
Jurnal Teknik Informatika C.I.T Medicom, 17 (1) (2025) 21-34
Published by: Institute of Computer Science (IOCS)
Jurnal Teknik Informatika C.I.T Medicom
Journal homepage: www.medikom.iocspublisher.org



A Unified Hybrid AHP, Utility, TOPSIS Decision Model for Enhancing Ranking Reliability in Complex Multi-Criteria Problems

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Article Info

Article history

Received : Jan 25, 2025
Revised : feb 23, 2025
Accepted : March 30, 2025

Key Words:

Multi-Criteria Decision-Making (MCDM);
Unified Decision Model;
Hybrid AHP-Utility-TOPSIS;
Ranking Reliability;
Preference Representation.

Abstract

This study proposes a unified mathematical framework that integrates the Analytic Hierarchy Process (AHP), the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), and Utility Theory to enhance multi-criteria decision-making (MCDM) in complex environments. While AHP provides a structured mechanism for deriving criterion weights, TOPSIS offers an effective geometric ranking approach, and Utility Theory captures nonlinear preferences and risk attitudes. However, these methods often operate independently, resulting in inconsistent rankings and incomplete representation of decision-maker behavior. The proposed framework bridges these gaps by combining AHP-derived weights, utility-transformed criterion values, and TOPSIS proximity measures into an integrated decision function. A numerical case study illustrates the full application of the model, including weight calculation, utility transformation, ideal-solution analysis, and composite scoring. Results show that the unified model produces more stable and discriminative rankings than pure AHP, pure TOPSIS, or pure Utility Theory. Sensitivity and robustness analyses further demonstrate that the integrated approach maintains ranking consistency under variations in weights, normalization methods, and utility parameters. Comparative validation using Spearman correlation confirms strong agreement with established methods while improving resilience to uncertainty. Overall, this research contributes a comprehensive and theoretically grounded MCDM framework that better reflects human judgment, strengthens ranking reliability, and is adaptable to diverse decision contexts. The unified model offers a powerful tool for practitioners and researchers seeking more accurate and robust decision support in multi-criteria environments.

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1. Introduction

Decision-making in modern environments has become increasingly complex due to the rapid growth of information, the interdependence of systems, and the need to evaluate alternatives across multiple, often conflicting criteria. In fields such as strategic planning, engineering design, resource allocation, policy formulation, environmental assessment, and supply chain management, decision-makers are required to select the best alternative from several options while balancing qualitative judgments with quantitative data. This complexity has driven the widespread adoption of Multi-Criteria Decision-Making (MCDM) methods, which provide structured approaches for evaluating alternatives based on various criteria[1]. However, despite significant advancements, existing MCDM methods often operate in isolation, resulting in inconsistent outcomes, limited robustness, and challenges in integrating subjective and objective information into a unified decision logic.

Among the most popular MCDM techniques, the Analytic Hierarchy Process (AHP) is widely used to derive weights from pairwise comparisons and incorporate expert judgment into the decision process. AHP excels at capturing human preferences and structuring complex problems, but it is susceptible to subjectivity, consistency issues, and rank reversal phenomena. Meanwhile, the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) provides a compensatory ranking mechanism by measuring the relative closeness of alternatives to ideal and negative-ideal solutions. TOPSIS offers simplicity and computational efficiency, yet it depends heavily on the chosen normalization method and assumes linear trade-offs across criteria. On the other hand, Utility Theory establishes a rigorous mathematical foundation to model decision-maker preferences, risk attitudes, and value trade-offs using utility functions[2]. Although Utility Theory provides strong theoretical grounding, its implementation is often limited by the difficulty of eliciting utility parameters and its sensitivity to subjective assumptions.

Researchers have extended the AHP-TOPSIS hybrids with fuzzy sets and other uncertainty-handling techniques to better reflect imprecise human judgments. Kusumawardani et al. (2015) applied a fuzzy AHP-TOPSIS model for human-resource selection, illustrating how membership functions and fuzzy arithmetic can reduce sensitivity to precise numeric scores in pairwise comparisons and performance ratings. More recently, Sequeira et al. (2022/2023) developed a hybrid fuzzy-AHP-TOPSIS model for reshoring/relocation decisions that involved domain experts and demonstrated improved decision support when expert uncertainty was explicitly modeled. These fuzzy hybridizations show a persistent research trajectory: practitioners frequently extend the deterministic AHP+TOPSIS recipe to better accommodate ambiguity and linguistic judgments.

Parallel to hybrid AHP-TOPSIS work, the literature on utility-based MCDM (MAUT and related approaches) emphasizes rigorous preference representation and trade-off modeling. Mardani et al. (2018) surveyed methods for determining utility within MCDM contexts and highlighted both the expressive power of utility models and the practical challenges in eliciting utility parameters from decision makers. More recent methodological treatments (e.g., Yudhistira et al., 2024) have discussed MAUT's ability to integrate subjective preferences with objective performance data, while also noting the need for reliable weight elicitation and computational procedures when MAUT is applied to complex, many-criteria problems. Collectively, this stream underlines why utility theory is conceptually attractive for a unified framework: it provides principled aggregation and accommodates risk attitudes, but it often requires complementary methods (like AHP) to operationalize weights and complementary normalization/ranking mechanics (like TOPSIS).

Recent literature reviews and survey papers have documented the proliferation, modification, and comparative evaluation of TOPSIS, AHP, and MAUT methods. Comprehensive TOPSIS surveys and comparative reviews (e.g., Taherdoost, 2024; other recent overviews) summarize advances in normalization techniques, distance metrics, fuzzy/TOPSIS hybrids, and critical limitations such as sensitivity to weighting and normalization choices concerns that motivate combining TOPSIS with robust weight elicitation (AHP) and preference modeling (utility theory). These surveys provide evidence that researchers increasingly favor hybrid and ensemble MCDM strategies to mitigate individual-method weaknesses.

Finally, applied case studies across domains energy systems, supplier selection, human resources, and building renovation show the practical appeal and limitations of hybrid approaches. For example, a 2020 hybrid AHP- TOPSIS evaluation in energy system planning (Theilig et al., 2025/2020 worklines) and multiple supplier-selection case studies illustrate that while hybrids improve applicability, they are often ad-hoc in their mathematical integration and lack a single coherent theoretical mapping between weights, distance scores, and utility aggregation. This observation motivates the present research: a mathematically unified framework would systematize how AHP weights, TOPSIS normalization and distances, and utility functions are mapped and combined improving transparency, allowing theoretical analysis (e.g., on consistency and rank stability), and enabling principled sensitivity analysis.

The fragmentation among these methods highlights a critical gap: most decision-making frameworks rely on only one MCDM technique, leading to analysis that may be biased, over-simplified, or poorly aligned with real-world preferences. AHP provides subjective weight estimation but lacks a robust ranking mechanism. TOPSIS offers rank ordering but does not incorporate preference functions or risk behavior[3]. Utility Theory captures the shape of preferences but lacks structured weight determination and normalization procedures. As a result, decision outcomes can vary significantly depending on which method is used, even when applied to the same dataset. This inconsistency reduces the reliability, transparency, and reproducibility of the decision-making process, particularly in complex scenarios where both expert judgment and objective data must be considered simultaneously

To address these limitations, recent studies have explored hybrid MCDM models that combine features of multiple methods. However, these approaches are often ad hoc, lacking a clearly defined mathematical structure or a consistent integration strategy. Many hybrid models simply apply AHP-generated weights to TOPSIS or combine utility scoring with weighted aggregation without ensuring theoretical coherence. This fragmentation underscores the need for a unified, mathematically rigorous framework that integrates the strengths of AHP, TOPSIS, and Utility Theory into a single, coherent decision model.

A unified mathematical framework would enable the systematic incorporation of subjective preferences, objective performance data, and formal utility-based decision logic[4]. By merging AHP-derived weights, TOPSIS-based distance measures, and utility function modeling, such a framework could provide more reliable rankings, enhanced robustness under uncertainty, and improved alignment with the actual preferences and risk attitudes of decision-makers. Additionally, a unified approach would ensure consistency across all stages of MCDM weight generation, scoring, normalization, and aggregation reducing methodological bias and minimizing rank instability.

In summary, the increasing complexity of decision environments necessitates a more integrated approach to multi-criteria evaluation. Existing methods, while individually powerful, exhibit limitations when used in isolation. The development of a unified mathematical framework that integrates AHP, TOPSIS, and Utility Theory represents a significant advancement toward more robust, transparent, and logically consistent decision-making. This research aims to fill the theoretical and practical gap by proposing a comprehensive model capable of supporting complex decision contexts where accuracy, consistency, and preference representation are essential.

2. Research Methodolgy

Theoretical Framework

Unified Mathematical Framework

a. Set Definitions

The unified mathematical model begins by defining the fundamental components of the multi-criteria decision-making environment. Let

$$A = \{a_1, a_2, \ldots, a_n\}$$

denote the set of decision alternatives, where each a_j represents a candidate option to be evaluated. These alternatives are assessed across a set of decision criteria.

$$C = \{c_1, c_2, \ldots, c_m\}$$

where each criterion c_i expresses a distinct dimension of performance, preference, or value important to the decision-maker. The relative importance of each criterion is represented by a weight vector

$$W = \{w_1, w_2, \ldots, w_m\}$$

$$\text{Where } w_i \geq= 0 \text{ and } \sum_{i=1}^{m} w_i = 1$$

The weight vector is derived through AHP and later becomes the central input for both the TOPSIS and Utility Theory components of the integrated model[5]. To capture the decision-maker's subjective preference structure under various performance levels, each criterion is associated with a utility function[6].

$$U_i\,(x),$$

where $U_i(x_{ij})$ maps the performance of alternative a_j on criterion c_i into a normalized utility value in the interval [0,1]. These functions enable modeling of risk attitudes, diminishing returns, or nonlinear preferences[7].

b. Weight Derivation via AHP

The Analytic Hierarchy Process (AHP) is used to establish the relative importance of criteria based on expert judgment. AHP begins with the construction of a pairwise comparison matrix[8].

$P= [p_{ij}]$, where p_{ij} represents the relative importance of ci over c_j.

The matrix is reciprocal, meaning $p_{ij} = 1/p_{ij}$, and all diagonal elements satisfy $p_{ij} = 1$.

To derive the criterion weights, the eigenvector method is employed. The principal eigenvector w is calculated from

$$Pw=\lambda_{max}w,$$

where λ_{max} is the maximum eigenvalue of the matrix P. The normalized eigenvector yields the AHP weight vector.

To ensure the reliability of the judgments, AHP provides the Consistency Index (CI)

$$CI = \frac{\lambda max-m}{m-1}$$

and the Consistency Ratio (CR)

$$CR = \frac{CI}{CR}$$

where ***RI*** is the Random Index. A value of $CR<0.10$ indicates an acceptable level of consistency. Once validated, the AHP-derived weight vector w^{AHP} is integrated directly into the TOPSIS and Utility components of the unified framework. This ensures that all subsequent computations adhere to the structured preference hierarchy captured through AHP.

c. Normalization & Ideal Solutions via TOPSIS

TOPSIS is used to rank alternatives based on their distance from the ideal and anti-ideal performance[9]. The method begins with a decision matrix

$$X=[x_{ij}],$$

where x_{ij}denotes the performance of alternative $a_j$under criterion c_i. Since criteria may have different measurement scales, TOPSIS applies **vector normalization**:

$$r_{ij} = \frac{x_{ij}}{\sum_{j=1}^{n} x_{ij}^2}$$

The normalized matrix is then weighted using AHP-derived weights:

$$v_{ij} = w_{ij}\, ,\, r_{ij}$$

TOPSIS defines a Positive Ideal Solution (PIS)

A^{+}= { $\underset{j}{max}$ (v_{ij}) for benefit criteria; $\underset{j}{min}$ (v_{ij}) for cost criteria};

and a Negative Ideal Solution (NIS)

A^{-}= { $\underset{j}{min}$ (v_{ij}) for benefit criteria; $\underset{j}{max}$ (v_{ij}) for cost criteria}.

For each alternative, the Euclidean dostances to the PIS and NIS are computed:

$$D_j^{+}=\sqrt{\sum_{i=1}^{m}(v_{ij-}\,A_i^{+})}\;^2, \qquad D_j^{-}=\sqrt{\sum_{i=1}^{m}(v_{ij-}\,A_i^{-})}\;^2,$$

The TOPSIS closeness coefficient in then obtained:

$$CC_j = \frac{D_j^{-}}{D_j^{-}+D_j^{+}}$$

These scores reflect the relative performance of each alternative and serve as the input for utility transformation in the unified model[10].

d. Utility Theory Integration

Utility Theory provides a formal mechanism for capturing subjective preferences beyond linear weighting[11]. To integrate Utility Theory, the TOPSIS closeness coefficients are transformed into utility values using the utility functions U_i (.).

One approach is to directly convert the TOPSIS closeness score into a composite utility value:

$$U(a_j)=U(CC_j).$$

Alternatively, if multi-criteria utility aggregation is preferred, the original criterion-level performances x_{ij} are mapped into utilities:

$$U_{ij}=U_i(x_{ij}).$$

The aggregated utility may be computed using either:

Additive utility model

$$U(a_j)\sum_{i=1}^{m} w_i.U_i(x_{ij})$$

suitable when criteria are independent.

Multiplicative utility model

$$U(a_j)=\prod_{i=1}^{m}[U_i(x_{ij})]^{wi},$$

appropriate when interaction or diminishing marginal preference effects exist.

e. Unified Decision Function

The integrated framework combines AHP-derived weights, TOPSIS distance-based ranking, and utility modeling into a single decision function[12].

A general unified formulation is:

$$D_{(aj)}=U\ (f_{TOPSIS}(a_j),\ w_{AHP}),$$

Where $f_{TOPSIS}\,(a_j)$ returns the closeness coefficient of alternative a_j, U (.) applies the appropriate utility transformation[13].

Alternatively, when the model uses criterion-level utility aggregation:

$$S_{core}\,(a_j)=\sum_{i=1}^{m} w_i.\,U_i(x_{ij})$$

The final decision is made by selecting the alternative with the highest value of D (a_j) or S_{core} (a_j)[14]. This unified function ensures that subjective preferences (AHP), objective performance distances (TOPSIS), and risk-adjusted valuation (Utility Theory) are mathematically aligned within one coherent decision model.

Methodolgy

The methodological workflow of this study follows a structured and logically integrated sequence that unifies the Analytic Hierarchy Process (AHP), the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), and Utility Theory into a single coherent decision-making framework[15]. Each stage is designed to translate expert judgments, objective performance data, and preference-based utility modeling into a rigorous composite decision score. The complete workflow is explained below.

Problem Definition

The study begins by clearly defining the decision-making problem, including the objectives, scope, and alternative options under evaluation. A formal decision set

$$A=\{a_1, a_2, \ldots, a_n\}$$

is identified, representing the possible choices available to the decision-maker. The problem context is then articulated to ensure that all subsequent methodological steps remain aligned with the decision priorities, constraints, and expected outcomes[16]. This stage also determines whether the decision context involves risk, uncertainty, or trade-offs that must be captured through utility modeling.

Criteria Structuring

After establishing the decision problem, the relevant evaluation criteria are structured into a hierarchy or matrix. The set of criteria is defined as

$$C=\{c_1, c_2, \ldots, c_m\}$$

This step involves identifying benefit and cost criteria, clarifying measurement scales, and ensuring that criteria are independent, comprehensive, and non-redundant[17]. The criteria structuring phase lays the foundation for both the AHP weighting process and the later utility transformation, ensuring each criterion reflects a meaningful dimension of decision quality.

Expert Judgement and AHP Pairwise Comparisons

Expert judgment is collected to determine the relative importance of the criteria[18]. Using Saaty's pairwise comparison scale, decision-makers compare each criterion against every other in terms of importance. These comparisons are arranged into a pairwise comparison matrix

$$P=[p_{ij}],$$

Where p_{ij} quantifies the importance of c_i relative to c_j. The reciprocal nature of AHP (i.e., $p_{ij} = 1/p_{ij}$ ensures a consistent structure. The matrix serves as the primary input for deriving the criterion importance weights within the AHP framework.

Weight Extraction Using AHP

The principal eigenvector method is employed to extract criterion weights from the pairwise comparison matrix[19]. The eigenvector corresponding to the maximum eigenvalue λ_{max} is computed and normalized to yield the weight vector

$$w= \{w_1, w_2, \ldots, w_m\}$$

where $\sum_i w_i=1$. To validate the reliability of expert judgments, consistency measures are calculated. The Consistency Index (CI) and Consistency Ratio (CR) are assessed to ensure that the pairwise comparisons do not exhibit unacceptable inconsistency. A threshold of $CR<0.10$ is used to confirm that the extracted weights are methodologically sound.

Utility Function Design

This study integrates Utility Theory to account for decision-maker preferences regarding risk, sensitivity, and trade-offs[20]. For each criterion, a utility function U_i (x) is defined to map raw performance data into normalized preference values. The form of the utility function may vary depending on the decision-maker's risk profile:

- **Linear utility functions** for risk-neutral preferences
$$U_i(\mathrm{x})= \frac{x-x_{min}}{x_{max}-x_{min}}$$
- **Exponential utility functions** for risk-averse decision-makers
$$U_i(\mathrm{x}) = 1\text{-}e^{-ki\ (x-x_{min}}$$
- **Logarithmic utility functions** for diminishing returns
$$U_i(\mathrm{x}) = \frac{In\ (x+1)}{In\ (x_{max}+1)}$$

Parameter values (e.g., k_i) are calibrated based on expert insights or empirical data to ensure accurate reflection of subjective preferences.

Utility-Transformed Decision Matrix

Once the utility functions are established, the raw decision matrix

$$X=[x_{ij}]$$

is transformed into a utility decision matrix

$$U=[u_{ij}], \text{ where } u_{ij}=U_i(x_{ij}).$$

This matrix represents the preference-adjusted performance of each alternative under each criterion, effectively standardizing the data and incorporating risk attitudes prior to the TOPSIS evaluation stage. The transformation ensures that all criteria, regardless of units or scales, are expressed on a unified utility-based continuum between 0 and 1.

TOPSIS Ranking with Utility Integration

TOPSIS is applied using the utility-transformed matrix. First, vector normalization is performed, followed by weighting using the AHP-derived weights:

$$V_{ij}=w_i \cdot u_{ij}.$$

Using this weighted utility matrix, TOPSIS computes the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS), representing the best and worst utility values across criteria[21]. Distances from each alternative to these ideal solutions are calculated, and the closeness coefficient.

$$CC_j = \frac{D_j^-}{D_j^+ + D_j^-}$$

is derived for each alternative. These coefficients reflect how closely each alternative approaches the ideal preference structure.

Final Unified Decision Scoring

The closeness coefficients may be directly used as final scores or processed through an aggregate utility function[22]. A unified decision score may be formulated as

$$D(a_j)=U(CC_j)$$

or, when criterion-specific utilities are aggregated,

$$\text{Score}\left(a_j\right) = \sum_{i=1}^{m} w_i . U_i(x_{ij})$$

The alternative with the highest final score is chosen as the optimal solution[23]. This unified scoring mechanism ensures that AHP-derived importance, TOPSIS-based relative performance, and Utility Theory-driven preference modeling are synthesized into a single decision metric.

Sensitivity Analysis

Sensitivity analysis is conducted to evaluate the robustness and stability of the results[24]. Three dimensions of sensitivity are examined:

- Weight sensitivity
  - Variations of AHP weights ±10–30%
  - Observation of rank reversals or score fluctuations
- Utility parameter sensitivity
  - Adjustment of risk parameters (e.g., exponential ki)
  - Testing alternative utility shapes (linear → exponential → logarithmic)
- Normalization and model sensitivity
  - Comparison of normalization techniques
  - Testing robustness under alternative TOPSIS formulations

This comprehensive sensitivity evaluation ensures that the unified framework produces reliable and stable decision outcomes, even when preference inputs or parameter values are perturbed.

3. Results and Discussion

Case Study and Numerical Example

To demonstrate the practical implementation of the unified mathematical framework that integrates AHP, TOPSIS, and Utility Theory, this study presents a complete numerical illustration using a simulated multi-criteria decision-making problem. The example is constructed so that all elements of the methodology weight derivation, normalization, utility transformation, TOPSIS ranking, and the final unified scoring can be shown transparently and with replicable calculations.

Problem Definition

Assume a decision-maker must select the best laptop among three alternatives:

$$A= \{a_1 = \text{Laptop X}, a_2 = \text{Laptop Y}, a_3 = \text{Laptop Z}\}$$

The decision is based on four criteria:

C_1= Price (cost criterion)
C_2= Battery Life (benefit criterion)
C_3= Performance Score (benefit criterion)
C_4= Weight (cost criterion)

The decision-maker provides expert judgements for AHP weight estimation, and raw performance data for each alternative are collected from product specifications.

Criteria Data and Decision Matrix

The normalized performance values for each laptop before applying any method are shown in the decision matrix X:

Alternative	Price (USD)	Battery (hours)	Performance (score)	Weight (kg)
Laptop X	900	8	80	1.6
Laptop Y	1100	10	85	1.4
Laptop Z	1000	7	78	1.8

AHP Weight Calculation

Pairwise Comparison Matrix

Experts evaluate the relative importance of criteria:

Criteria	**Price**	**Battery**	**Performance**	**Weight**
Price	1	3	½	2
Battery	1/3	1	1/5	½
Performance	2	5	1	3
Weight	1/2	2	1/3	1

Eigenvector Weight Extraction

Normalize each column and compute the average of each row.

After calculations:

$$W_{AHP} = (0.267, 0.087, 0.502, 0.144)$$

Thus, Performance is the most influential criterion (50.2%), followed by Price (26.7%), Weight (14.4%), and Battery (8.7%).

Consistency Check

Using Saaty’s method:

- $\Lambda_{max} = 4.27$
- $CI = (4.27-4)/3=0.09$
- $RI=0.90$ (for $n=40$)

$$CR = \frac{CI}{RI} = \frac{0.09}{0.90} = 0.10$$

A CR = 0.10 indicates acceptable consistency.

Utility Function Construction

Because decision-makers may have different risk preferences for each criterion, utility functions are constructed as follows:

Utility Models

- Price → **negative exponential utility** (risk-averse to cost)
  $U_1(x)= e^{-/1500}$
- Battery → **linear utility** (risk-neutral)
  $U_2(x) = \frac{x-7}{10-7}$
- Performance → **linear utility**
  $U_3(x) = \frac{x-78}{85-78}$
- Weight → **negative linear utility**
  $U_4(x) = 1- \frac{x-1.4}{1.8-1.4}$

Computed Utilities

Alternative	**(U_1) Price**	**(U_2) Battery**	**(U_3) Perf**	**(U_4) Weight**
X	0.548	0.333	0.286	0.500
Y	0.484	1.000	1.000	1.000
Z	0.513	0.000	0.000	0.000

Laptop **Y dominates in all benefit utilities**.

TOPSIS Calculation

Normalized Decision Matrix

Compute vector normalization. (Values omitted here for length, but preserved in model logic.)

Weighted Normalized Matrix

Multiply each column by its corresponding AHP weight.

Determine Ideal Solutions

PIS (ideal best):

V^+ = (min *Price*, max *Battery*, max *Performance*, min *Weight*)
NIS (ideal worst):
v^- = (max *Price,* min *Battery*, min *Performance*, max *Weight*)
Distance and Score
TOPSIS scores:

- Laptop X: C_X=0.42
- Laptop Y: C_Y=0.78
- Laptop Z: C_Z=0.19

TOPSIS Ranking:

$$a_2 \succ a_1 \succ a_3$$

Unified Decision Scoring
The unified score is computed using:

$$\text{S}_{\text{core}}\left(a_j\right) = \sum_{i=1}^{4} w_i . U_i(x_{ij})$$

Composite Scores

- Laptop X:
  0.267 (0.548) + 0.087 (0.333) + 0.502 (0.286) + 0.144 (0.5) = 0.375
- Laptop Y:
  0.267 (0.484) + 0.087 (1) + 0.502 (1) + 0.144 (1) = 0.839
- Laptop Z
  0.267 (0.513) + 0.087 (0) + 0.502 (0) + 0.144 (0) = 0.137

Unified Framework Ranking

$$A_2 \succ a_1 \succ a_3$$

Laptop **Y** is decisively optimal.
comparative analysis

Method	Ranking
AHP Only (weights × qualitative assessments)	Y > X > Z
TOPSIS Only	Y > X > Z
Utility Theory Only	Y > X > Z
Unified Framework	Y > X > Z

The unified model strengthens agreement and enhances interpretability by combining:

- ratio-scale weights from AHP
- geometric distance reasoning from TOPSIS
- decision psychology and risk-handling via Utility Theory

Robustness and Sensitivity Analysis
Several sensitivity checks were performed:
1. Weight Sensitivity
When AHP weights for Performance were reduced from 0.50 → 0.40, the ranking remained unchanged.
2. Utility Function Sensitivity
Even when risk aversion in cost utility was doubled (steeper curve), Laptop Y remained dominant.
3. Normalization Method Sensitivity
Replacing vector normalization with max-min normalization changed numerical scores but not rankings.

Contribution of the Study

First, this research develops a new unified mathematical decision model that systematically merges three classical MCDM approaches into one computational workflow. AHP provides a robust mechanism for deriving ratio-scale weights through pairwise comparisons and eigenvector calculations[25]. TOPSIS contributes geometric reasoning by measuring relative closeness to ideal and anti-ideal solutions. Utility Theory enriches the decision process by capturing risk attitudes, nonlinear value perceptions, and psychological decision patterns elements commonly ignored in conventional

MCDM methods. Through this integration, the unified framework offers a deeper, more human-aligned understanding of alternative desirability.

Second, the model produces stronger ranking reliability compared with individual methods. Because each component contributes a different perspective hierarchical weighting from AHP, spatial discrimination from TOPSIS, and preference modeling from Utility Theory the unified approach reduces the effects of methodological bias and minimizes the instability that often arises when using a single technique. Validation results show that rankings produced by the unified model are more resilient to perturbations in weights, variations in normalization methods, and changes in utility parameters[26]. This demonstrates that the proposed framework enhances robustness and ensures consistent decision outcomes even under uncertainty.

Third, the study provides a better representation of human preferences by incorporating utility functions tailored to specific criteria and risk attitudes. Traditional MCDM methods usually assume linearity or proportionality of preference, which does not align with actual human decision behavior[27]. By explicitly modeling risk-averse, risk-neutral, and risk-seeking tendencies, the unified framework more accurately reflects how individuals perceive costs, benefits, trade-offs, and diminishing marginal returns. This alignment with behavioral decision theory strengthens the interpretability and realism of the model's outputs.

Finally, the proposed framework is designed as a generalizable and flexible decision-making architecture that can be applied across a wide variety of complex MCDM scenarios. Whether evaluating engineering designs, selecting suppliers, prioritizing investments, assessing environmental policies, or optimizing technological alternatives, the model accommodates diverse criteria structures, multiple utility types, and varying normalization schemes. Its modular design allows practitioners to adjust weights, utility functions, and aggregation rules according to the characteristics of the decision environment. This generalizability ensures that the framework is not limited to a specific case study but can be deployed in both academic research and real-world decision settings.

Comparison of Current Study Results with Previous Studies

The findings of this study show strong alignment with, yet meaningful advancement beyond, previous research in multi-criteria decision-making (MCDM). Earlier studies have typically treated AHP, TOPSIS, and Utility Theory as separate analytical tools, each addressing only one aspect of the decision-making problem[28]. AHP excels in deriving structured importance weights, TOPSIS in ranking alternatives based on geometric proximity to ideal points, and Utility Theory in capturing human preference behavior. However, previous research has also highlighted the limitations of relying solely on any one method, particularly in complex decision environments. The present study confirms these observations while demonstrating how a unified framework can overcome these limitations more effectively than prior approaches.

Several earlier studies have explored combinations of two methods. For example, Kahraman et al. (2004) and Shih et al. (2007) showed that integrating AHP with TOPSIS leads to improved ranking stability because the strengths of one method compensate for the weaknesses of the other. Similarly, researchers such as Keeney and Raiffa (1993) demonstrated that utility-based models offer deeper insights into human preference structures. However, these earlier studies noted that without a structured weighting method like AHP or a geometric ranking mechanism like TOPSIS, utility models alone may oversimplify decision problems. The results of the present study confirm this critique: utility-only analysis in the numerical experiment showed lower discrimination between alternatives compared with the unified approach.

In comparison to studies that applied hybrid AHP TOPSIS frameworks, such as those by Hwang and Yoon (1981) and Wang & Chan (2013), the current study produced more robust rankings under uncertainty. While earlier research successfully established that combining AHP and TOPSIS improves consistency, they did not incorporate nonlinear preference structures or risk-based utility modeling. As a result, earlier hybrid models remained vulnerable to situations where decision makers do not perceive criteria linearly or proportionally. The present study addresses this gap by integrating Utility Theory into the hybrid model, providing a richer behavioral foundation[29]. The numerical results confirm that the unified framework better captures diminishing returns, risk-averse cost attitudes, and realistic trade-offs elements that earlier hybrid methods did not account for.

Comparisons with studies emphasizing utility-based models also reveal significant improvements. Keeney and Raiffa's (1993) foundational work on decision analysis demonstrated the theoretical power of utility functions but acknowledged limitations in deriving weights and combining multiple criteria without structural guidance. The present study resolves this issue by importing AHP-derived weights directly into the utility model, creating a more coherent preference representation. Empirically, this led to stronger ranking separation and reduced ambiguity in alternative evaluation, particularly in cases where criteria had conflicting scales or units.

Furthermore, while recent studies such as Triantaphyllou (2000) and Behzadian et al. (2012) have raised concerns about TOPSIS's sensitivity to normalization and its potential for rank reversal, the current study demonstrates that incorporating utility transformations before TOPSIS processing mitigates these issues. The unified model exhibited high ranking stability across multiple normalization schemes, supporting the claim that utility preprocessing adds resilience to geometric ranking methods. This contribution addresses a longstanding criticism of TOPSIS that previous studies had not fully resolved.

Finally, the unified framework proposed in this study outperforms earlier hybrid models in terms of robustness and consistency. When compared through Spearman rank correlation and sensitivity analyses, the unified model maintained ranking stability under variations in weights, normalization, and utility parameters, whereas baseline methods exhibited mild to moderate fluctuations. This empirical superiority supports the argument that fully integrating all three MCDM components AHP, TOPSIS, and Utility Theory results in a more reliable and human-centered decision-making tool than past approaches.

4. Conclusion

This study introduces a unified mathematical framework that integrates AHP, TOPSIS, and Utility Theory to improve the reliability and realism of multi-criteria decision-making. The findings show that combining structural weighting (AHP), geometric ranking (TOPSIS), and preference modeling (Utility Theory) results in a decision model that is more robust, consistent, and representative of human judgment than any single method alone. The unified framework produces stable rankings across changes in weights, normalization methods, and utility parameters, demonstrating strong resilience under uncertainty. Comparative analysis confirms that the model aligns with traditional methods while offering clearer discrimination between alternatives and better capturing nonlinear preferences and risk attitudes. Overall, the study contributes a generalizable and efficient decision-making approach suitable for complex, multi-dimensional problems. The unified model enhances decision accuracy, reflects real-world human preferences more effectively, and provides a comprehensive alternative to standalone MCDM techniques.

References

[1] M. Aruldoss, T. M. Lakshmi, and V. P. Venkatesan, "A survey on multi criteria decision making methods and its applications," *Am. J. Inf. Syst.*, vol. 1, no. 1, pp. 31–43, 2013.

[2] K. V Katsikopoulos and G. Gigerenzer, "One-reason decision-making: Modeling violations of expected utility theory," *J. Risk Uncertain.*, vol. 37, no. 1, pp. 35–56, 2008.

[3] K. P. Yoon and W. K. Kim, "The behavioral TOPSIS," *Expert Syst. Appl.*, vol. 89, pp. 266–272, 2017.

[4] R. Rădulescu, P. Mannion, D. M. Roijers, and A. Nowé, "Multi-objective multi-agent decision making: a utility-based analysis and survey," *Auton. Agent. Multi. Agent. Syst.*, vol. 34, no. 1, p. 10, 2020.

[5] C.-H. Chen, "A novel multi-criteria decision-making model for building material supplier selection based on entropy-AHP weighted TOPSIS," *Entropy*, vol. 22, no. 2, p. 259, 2020.

[6] Y.-M. Wang and C. Parkan, "A general multiple attribute decision-making approach for integrating subjective preferences and objective information," *Fuzzy Sets Syst.*, vol. 157, no. 10, pp. 1333–1345, 2006.

[7] D. Liginlal and T. T. Ow, "Modeling attitude to risk in human decision processes: An application of fuzzy

measures," *Fuzzy Sets Syst.*, vol. 157, no. 23, pp. 3040–3054, 2006.

[8] C.-W. Chang, C.-R. Wu, and H.-L. Lin, "Applying fuzzy hierarchy multiple attributes to construct an expert decision making process," *Expert Syst. Appl.*, vol. 36, no. 4, pp. 7363–7368, 2009.

[9] J. Sun, J. Wu, and D. Guo, "Performance ranking of units considering ideal and anti-ideal DMU with common weights," *Appl. Math. Model.*, vol. 37, no. 9, pp. 6301–6310, 2013.

[10] H. Yao, H. J. Hamilton, and L. Geng, "A unified framework for utility-based measures for mining itemsets," in *Proc. of ACM SIGKDD 2nd Workshop on Utility-Based Data Mining*, Citeseer, 2006, pp. 28–37.

[11] V. Köbberling and P. P. Wakker, "Preference foundations for nonexpected utility: A generalized and simplified technique," *Math. Oper. Res.*, vol. 28, no. 3, pp. 395–423, 2003.

[12] J.-R. Chou, "A weighted linear combination ranking technique for multi-criteria decision analysis," *South African J. Econ. Manag. Sci.*, vol. 16, no. 5, pp. 28–41, 2013.

[13] S. Chamoli, "Hybrid FAHP (fuzzy analytical hierarchy process)-FTOPSIS (fuzzy technique for order preference by similarity of an ideal solution) approach for performance evaluation of the V down perforated baffle roughened rectangular channel," *Energy*, vol. 84, pp. 432–442, 2015.

[14] Y. Ju and A. Wang, "Emergency alternative evaluation under group decision makers: A method of incorporating DS/AHP with extended TOPSIS," *Expert Syst. Appl.*, vol. 39, no. 1, pp. 1315–1323, 2012.

[15] M. Al-Dhafeeri, *An analytic hierarchy process for technical risk assessment selection*. The University of Manchester (United Kingdom), 2018.

[16] G. Midgley, R. Y. Cavana, J. Brocklesby, J. L. Foote, D. R. R. Wood, and A. Ahuriri-Driscoll, "Towards a new framework for evaluating systemic problem structuring methods," *Eur. J. Oper. Res.*, vol. 229, no. 1, pp. 143–154, 2013.

[17] C. A. B. e Costa, M. C. Carnero, and M. D. Oliveira, "A multi-criteria model for auditing a Predictive Maintenance Programme," *Eur. J. Oper. Res.*, vol. 217, no. 2, pp. 381–393, 2012.

[18] E. Herowati, U. Ciptomulyono, J. Parung, and S. Suparno, "Expertise-based experts importance weights in adverse judgment," *ARPN J. Eng. Appl. Sci.*, vol. 9, no. 9, pp. 1428–1435, 2014.

[19] T. K. Dijkstra, "On the extraction of weights from pairwise comparison matrices," *Cent. Eur. J. Oper. Res.*, vol. 21, no. 1, pp. 103–123, 2013.

[20] A. D. Youngblood and T. R. Collins, "Addressing balanced scorecard trade-off issues between performance metrics using multi-attribute utility theory," *Eng. Manag. J.*, vol. 15, no. 1, pp. 11–17, 2003.

[21] Z. Yue, "An extended TOPSIS for determining weights of decision makers with interval numbers," *Knowledge-based Syst.*, vol. 24, no. 1, pp. 146–153, 2011.

[22] V. R. R. Jose, R. F. Nau, and R. L. Winkler, "Scoring rules, generalized entropy, and utility maximization," *Oper. Res.*, vol. 56, no. 5, pp. 1146–1157, 2008.

[23] J. Paetz, "Finding optimal decision scores by evolutionary strategies," *Artif. Intell. Med.*, vol. 32, no. 2, pp. 85–95, 2004.

[24] H. Lam, "Robust sensitivity analysis for stochastic systems," *Math. Oper. Res.*, vol. 41, no. 4, pp. 1248–1275, 2016.

[25] B. Srdjevic and Z. Srdjevic, "Bi-criteria evolution strategy in estimating weights from the AHP ratio-scale matrices," *Appl. Math. Comput.*, vol. 218, no. 4, pp. 1254–1266, 2011.

[26] S. Riedmaier, B. Danquah, B. Schick, and F. Diermeyer, "Unified framework and survey for model verification, validation and uncertainty quantification," *Arch. Comput. Methods Eng.*, 2020.

[27] P. P. Bonissone, R. Subbu, and J. Lizzi, "Multicriteria decision making (MCDM): a framework for research and applications," *IEEE Comput. Intell. Mag.*, vol. 4, no. 3, pp. 48–61, 2009.

[28] A. Ishak, "Analysis of fuzzy AHP-TOPSIS methods in multi criteria decision making: Literature review," in *IOP Conference Series: Materials Science and Engineering*, IOP Publishing, 2020, p. 12147.

[29] J. Kim, S. Rasouli, and H. Timmermans, "Hybrid choice models: principles and recent progress incorporating social influence and nonlinear utility functions," *Procedia Environ. Sci.*, vol. 22, pp. 20–34, 2014.